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CORRELATIONS AND BEAM NOISE

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1. Introduction

The time-varying currents generated by a particle beam in a storage ring produce rf electromagnetic fields which can be intercepted by a pickup structure for beam diagnostics. The temporal variation of the current has several sources: macroscopic bunching, collective oscillations, and the underlying discrete nature of a particle beam. This discreteness at the microscopic level provides the high-frequency structure necessary for the production of Schottky noise, which is readily observable at rf frequencies, and for the emission of incoherent synchrotron radiation at yet higher frequencies. It is not, however, the discreteness alone, but rather the combination of discreteness and randomness in position of the particles relative to each other that is responsible for the typical spectral character of the noise. If this randomness is not complete—that is, if the particle positions are correlated—the rf signal can be significantly modified and as a function of frequency can be enhanced or suppressed. It is the nature of these correlations and their detailed effects on Schottky noise that are the concern of this paper.

The development of microscopic correlations is driven by interaction between the beam particles. A primary example of such an interaction is the long-range Coulomb force for a collection of many charged particles. This system, of course, is the area of study of plasma physics where many powerful physical concepts and mathematical tools have been developed. One such notion is that of Debye screening, where the field of any test charge is shielded at long distances by the redistribution of neighboring particles in response to the Coulomb field of that test charge. This microscopic ordering of the particles in space (in other words, the development of correlations between the particle positions) does not change the macroscopic average distribution. Yet the dynamics of the plasma and the interparticle coupling are strongly affected.

An accelerator beam is presented with a considerably more complicated electromagnetic environment than a simple, unbounded plasma. Interaction between the beam particles can arise through coupling to the vacuum chamber wall and through the electromagnetic fields generated by beam excitation of cavities, bellows, and other discontinuities. In addition to these passive structures, active feedback systems using localized pickups and kickers for damping instabilities and cooling stochastically can induce coupling between particles, and given electronic time delays, dissipative self-interaction. Although the time and position (or frequency and wavenumber) dependence of these interactions is quite different from the free-space Coulomb interaction, they share with it and with each other a long-range character because of the limited bandwidth of rf structures. Therefore, these systems can be analyzed with many of the tools and techniques of plasma physics. On the other hand, these forces do differ from free-space electromagnetics in having dissipation, less singular small-distance behavior (faster high-frequency rolloff), and the lack of translation invariance (localized structures).

It is the intent of this paper to present (with the clear exception of Section 5) a heuristic view of correlation phenomena in particle beams, and to stress the generality of the analysis to most long-range particle interactions of moderate strength. The reader is urged to refer to earlier works in this accelerator school series. Specifically, Bisognano and Leemann [1] presents a generalized BBGKY approach for dissipative systems with applications to stochastic cooling. Bisognano [2] presents some general concepts of noise theory and stochastic processes. Chattopadhyay [3] is also of considerable interest.

2. Basics of Schottky Noise

The current produced by a point particle has a flat spectral content extending to infinite frequency. In a storage ring this current repeats indefinitely, producing at any location a train of pulses. There can be constructive or destructive interference between the spectra of these pulses (because of the time delay between them) which yields a variation in the overall spectrum observed. We proceed with a detailed analysis of tranverse Schottky noise; the description of the longitudinal noise is described elsewhere [1, 2], and follows a similar chain of reasoning.

The dipole current produced at a pickup located at a particular azimuth in a storage ring by a particle i undergoing betatron oscillations is of the form

$$I_i(t) = ea_i \sum_{n=-\infty}^{+\infty} \cos(\nu_i\omega_i t + \phi_i)\, \delta\,(t - t_{0i} - nT_i) \tag{2–1}$$

where $f_i = \frac{\omega_i}{2\pi} = \frac{1}{T_i}$ is the revolution frequency, a_i is the oscillation amplitude, and ν_i is the betatron tune. The parameters ϕ_i and t_{0i} establish an arbitrary phase and an arbitrary transit time, respectively, for the particle. On using the identity

$$\sum_{n=-\infty}^{+\infty} \delta(t - nT_i) = f_i \sum_{n=-\infty}^{+\infty} e^{in\omega_i t} \tag{2–2}$$

which follows from the periodic nature of the delta-function train, we have

$$I_i(t) = \frac{ea_i f_i}{2} \sum_{\pm} \sum_{n=-\infty}^{+\infty} e^{\pm i\nu_i\omega_i t}\, e^{\pm i\phi_i}\, e^{in\omega_i(t-t_{0i})} \tag{2–3}$$

The point nature of the source is manifest in the extension of the spectrum to infinity; the periodic nature of the source is responsible for the line structure at harmonics $(n \pm \nu_i)\omega_i$. We have implicitly assumed that the a_i and f_i are independent of time.

Now consider a collection of N particles in the ring. We sum Eq. (2-3) over the particle label i and find that the total dipole current is

$$I_d\,(t) \;=\; \sum_{i=1}^{N}\sum_{\pm}\sum_{n=-\infty}^{+\infty} \frac{ea_i f_i}{2}\; e^{\pm i\nu_i\omega_i t}\; e^{\pm i\phi_i}\; e^{in\omega_i(t-t_{oi})} \tag{2–4}$$

For each harmonic n the dipole current at the pickup is composed of two betatron sidebands corresponding to the range of $(n \pm \nu_i)\omega_i$. It is possible for the sidebands to overlap (Schottky band overlap) if $(n \pm \nu_i)\omega_i \;=\; (m \pm \nu_j)\omega_j$ for some i and j, and $(m \pm \nu) \;\neq\; (n \pm \nu)$.

3. The Power Spectrum of Transverse Schottky Noise

The dipole current will produce a signal in a transverse pickup which can be amplified and processed. This signal will generally be of the form

$$s(t) = \sum_{i=1}^{N}\sum_{\pm}\sum_{n=-\infty}^{+\infty} \frac{ea_i f_i}{2}\; G\,(n\omega_i \pm \nu_i\omega_i)\; e^{i(n\pm\nu_i)\omega_i t}\; e^{\pm i\phi_i}\; e^{in\omega_i t_{oi}} \tag{3–1}$$

after transients have settled down, and where $G(\Omega)$ represents the electronic transfer character of the diagnostic system at electronic frequency (that is, the Fourier conjugate of time) Ω. Typically G will roll off rapidly at high frequencies and the sum will converge absolutely. The bandwidth of the gain G can cover several Schottky bands or can be a fraction of a single band. For example, if the structure of a single Schottky sideband is desired, the bandwidth of G must be small compared to the width of that Schottky band. Given this noise source $s(t)$, we can study its power spectrum with Fourier analysis. The duration of a measurement is clearly finite, say $2T$. We define the average power in this noise source by

$$P_{\rm av} \;=\; \frac{1}{2T}\int_{-T}^{+T} dt\; |s(t)|^2 \tag{3–2}$$

On inserting Eq. (3-1) into Eq. (3-2) and performing the integration we have

$$\begin{aligned} P_{\rm av} \;=\; &\sum_{i}\sum_{j}\sum_{n}\sum_{m}\sum_{\pm}\sum_{\oplus\ominus} G(n\omega_i \pm \nu_i\omega_i)\; G^*(m\omega_j \oplus \ominus\nu_j\omega_j) \\ &\times \frac{\sin\Big((n\omega_i \pm \nu\omega_i) \;-\; (m\omega_j \oplus \ominus\nu\omega_j)\Big)T}{\Big((n\omega_i \pm \nu\omega_i) - (m\omega_j \oplus \ominus\nu\omega_j)\Big)T} \\ &\times \frac{1}{4}e^2 a_i a_j f_i f_j\; e^{\pm i\phi_i}\; e^{\oplus\ominus i\phi_j}\; e^{in\omega_i t_{oi}}\; e^{-im\omega_j t_{oj}} \end{aligned} \tag{3–3}$$

where the $\pm$ and $\oplus\ominus$ sums are independent.

Note that for finite times T, frequencies within $1/T$ can interfere strongly with each other through the sine factor. This is an example of the uncertainty principle for Fourier transforms. For this argument we assume that (bandwidth of G) $\gg \frac{1}{T}$. For any finite sampling time the

Schottky power can be strongly affected by particle-to-particle correlations which can produce interference through the phase factors.

As will be seen in Section 5, the correlations induced through the beam environment actually exhibit quite singular (delta-function-like) behavior in frequency. In other words, if we observe the beam after transients have settled down, the beam has always had longer to respond to particle-particle interactions than we have had to observe them. Thus, the ability of the beam to produce small frequency resolution correlations in general exceeds the experimenter's measurement resolution. That is, we expect the bulk of correlations to occur for frequency differences much less than $\frac{1}{T}$.

A few examples of the effects of correlation are now in order. First, consider the case of statistically independent particles in a uniform, coasting beam. The particle positions are totally random with respect to each other, and all interparticle phases are uncorrelated. When averaged over independent random distributions the cross terms of the sum in Eq. (3-3) vanish, both between coordinates of different particles and between coordinates of the same particle for different $n \pm \nu$ and $m \pm \nu$ (since $\omega_i t_{0i}$ extends over all phases), and the double sum over i and j collapses to a single sum. The power is found to be proportional to the number of particles with betatron harmonics in the bandpass of the gain G. For (bandwidth G) $<$ (Schottky bandwidth), we have a determination of the single-particle frequency distribution. In other words, the Schottky noise distribution mimics the single-particle revolution frequency distribution. Second, consider a coherent betatron oscillation on a coasting beam and assume that the bandwidth of G covers a single sideband. Suppose the beam is offset initially at $t = -T$ with a betatron phase $\phi = n\theta$ where the angle θ is measured with respect to the pickup structure. For $T \ll$ (spread in $n\omega_i)^{-1}$ this phase will approximately cancel the $n\omega_i t_{0i}$ term for the full duration of the integration in Eq. (3-2). The signal of all particles will add coherently and the resultant power will be proportional to the square of the number of particles within the frequency resolution of the diagnostic system. Without self-interaction, this coherence would vanish if instead $T \gg$ (spread in $n\omega_i)^{-1}$ because the phase cancellation will be degraded by the frequency differences. This will be discussed more fully in subsequent sections. Third, consider a short bunch oscillating in a rigid dipole oscillation. The betatron phases are identical and for small enough n such that all differences $n(\phi_i - \phi_j) \ll 2\pi$, the particle signals are again essentially in phase, and their resultant signal for a single sideband is again proportional to the square of the number of particles. At frequencies sufficiently high so that $n(\phi_i - \phi_j) \gtrsim 2\pi$, the enhanced signal disappears as the phase differences average away the cross terms. Also note that for $n\phi_i - m\phi_j \ll 2\pi$ there can be interference effects between neighboring bands, if they fall within the bandwidth of G. This is the result of the bunching; in more sophisticated terms, bunched-beam Schottky noise is nonstationary (it depends on time; sometimes it's on, sometimes it's off). For nonstationary noise, the noise amplitudes in the neighborhood of two different frequencies are not statistically independent.

In each of the last two cases, the signal was enhanced in some band due to an ordering (correlations) of the particle positions at a macroscopic level. Schottky noise can also be diminished from its random-phase value and the correlation can be microscopic in character. Clearly, from Eq. (3-3), for Schottky noise suppression the sum of the betatron oscillations of the particles of the sample must be near zero; that is, the average transverse position of the sample must be small.

Consider a sample of N particles taken from a distribution with average transverse position zero and rms position σ. From random fluctuations the average position of the sample will be distributed with an rms value of $\frac{\sigma}{\sqrt{N}}$. For a relatively large sample of particles, the average position can be made zero by shifting the individual particle coordinate by a value of order $\frac{\sigma}{\sqrt{N}}$, which is small compared to the beam size σ for a large number of particles. For such a system, typical of a centered particle beam in a transport channel, the Schottky signal can be made to disappear by a microscopic ordering of the particles which does not significantly change the macroscopic distribution.

Before proceeding with our investigation of Schottky noise suppression, let us take one more look at Eq. (3-3). Let the N particles be distributed in frequency with a full-width $\Delta\Omega$ and let $T \gg \frac{N}{\Delta\Omega}$. Then the sine factor will be negligible except in the unlikely event that two particles are much closer in frequency than their average spacing. It would appear that after a long enough time the correlational effects would essentially disappear from the Schottky diagnostic or would become very susceptible to the fluctuations in particle distribution. The system would be able to sort itself through frequency differences. Section 5 describes correlational effects through a dielectric model; that is, the shielding a test charge experiences due to correlations with other beam particles will be approximated by polarization of a continuous medium. Such a model is at the heart of much of plasma theory. How can this continuum model be reconciled with the discreteness which should be apparent after long times? The answer lies in the stability of the revolution frequency distribution at the microscopic level. For example, consider the action of stochastic cooling, the damping of betatron oscillations or momentum spread of a particle beam by a feedback system. In computer modeling of transverse stochastic cooling systems with randomly distributed but fixed revolution frequencies, cooling rates are indeed found to degrade after times long enough to resolve the fine structure of the frequency distribution. This degradation is seen to depend on the details of frequency differences at the particle-to-particle level. When a small jitter in longitudinal frequency is added to the model, this effect disappears. Modeling of longitudinal cooling does not exhibit this problem because of the implicit jitter introduced in the process of diminishing energy spread. When these effects are scaled to actual accelerator systems, energy stability of the order of electron volts for tens of seconds would be required in a GeV beam being cooled at a rate of MeV per second. Such stability is not to be expected; however, it should be noted that some microscopic jitter is required to justify much of the successful mathematical analysis in the literature.

4. A Simple Example

In the previous section we have seen that an interaction which acts to set the average position of a sample of the beam to zero can diminish the Schottky noise in a given frequency band. Such an arrangement is found in transverse dampers for instabilities and stochastic cooling. However, there is a second mechanism that disrupts achievement of this perfect cancellation—the betatron oscillators do not have exactly the same oscillation frequency. If the average of the oscillators is set to zero, the frequency difference will cause a reemergence of a nonzero average. Before proceeding in Section 5 with a thorough but mathematically complex analysis, we will first study a simple system that can be solved in closed form to highlight the physical mechanisms and, in particular, the various time scales involved.

Consider a set of 10 harmonic oscillators with frequencies $\Omega_i = 1.001, 1.002, ..., 1.010$; i.e., the interparticle frequency spacing δ is 0.001. Let there be a feedback system which detects the average $\dot{x}$ of the system (this is basically what a transverse stochastic cooling system does by detection of x and application of the amplified signal a quarter betatron wavelength downstream) and applies it back on the 10 oscillators. The equations of motion for this system are

$$\ddot{x}_i + \Omega_i^2 x_i = -g \sum_{j=1}^{10} \dot{x}_j \text{ for all } i \tag{4–1}$$

We now study this well-defined eigenvalue problem as a function of the parameter g. Assume solutions are of the form $e^{\alpha t}$. Fig. 4-1 is a plot of the real and positive imaginary values of the ten eigenfrequencies α for $g = 0$. There is, of course, a conjugate set of eigenvalues with negative imaginary parts. Fig. 4-2 corresponds to $g = 0.0002$. Note that the real part each of the ten eigenfrequencies is approximately $g/2$. Each oscillator now has damped motion corresponding to coupling to its own velocity alone, and the effects of the other oscillators are negligible.

Each oscillator behaves independently of the others because their frequency differences prevent long-term coherence. As g is further increased this decoherence is no longer in force, and the eigenvalue plot is deformed (see Figures 4-3 to 4-6) until (Fig. 4-7) at $g = 0.005$ a single eigenvalue has split off with a real part of $Ng/2$. This fast rate corresponds to total coherence of the oscillators in this eigenmode.

The remaining 9 eigenvalues have fallen back to virtually pure imaginary values. The associated 9 eigenvectors share in one common property:

$$\sum_{i=1}^{10} \dot{x}_i \approx 0 \tag{4–2}$$

But when condition (4-2) is satisfied

$$\left(\sum_{i=1}^{10} \dot{x}_i \right)^2 \approx 0 \tag{4–3}$$

and

$$\sum_{i \neq j} \dot{x}_i \dot{x}_j \approx - \sum_i \dot{x}_i^2 < 0 \tag{4–4}$$

The damping system has damped out the average velocites of the system. When the average velocity is zero, the correction signal goes away, and damping stops. From Eq. (4-4) we see

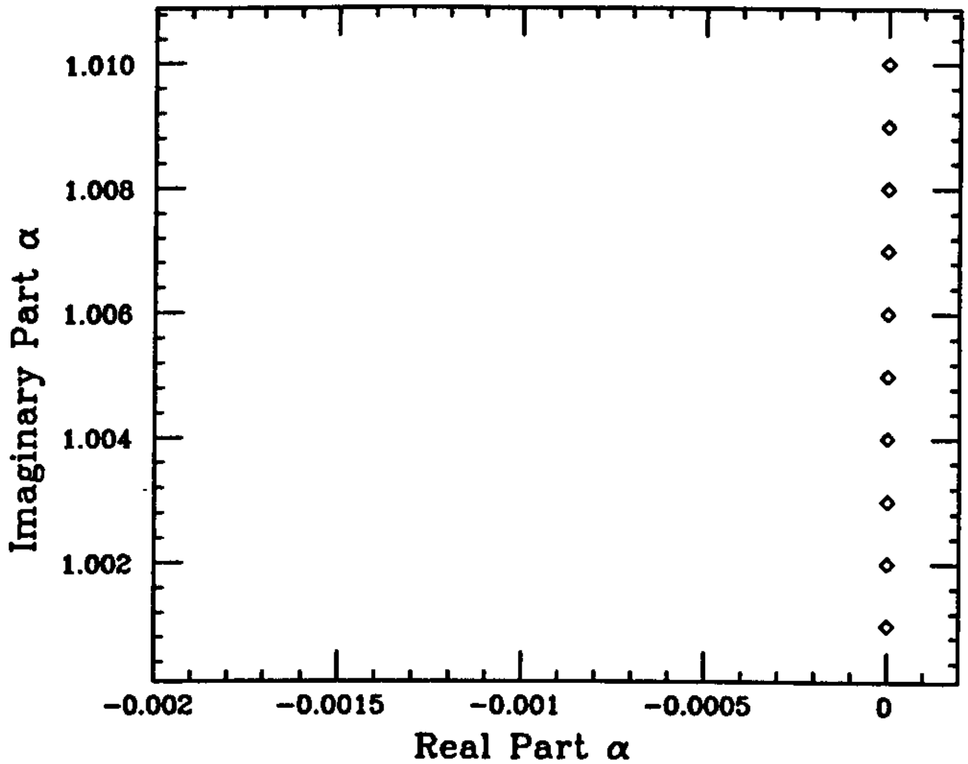

Figure 4-1 Complex frequency spectrum for g=0.0.

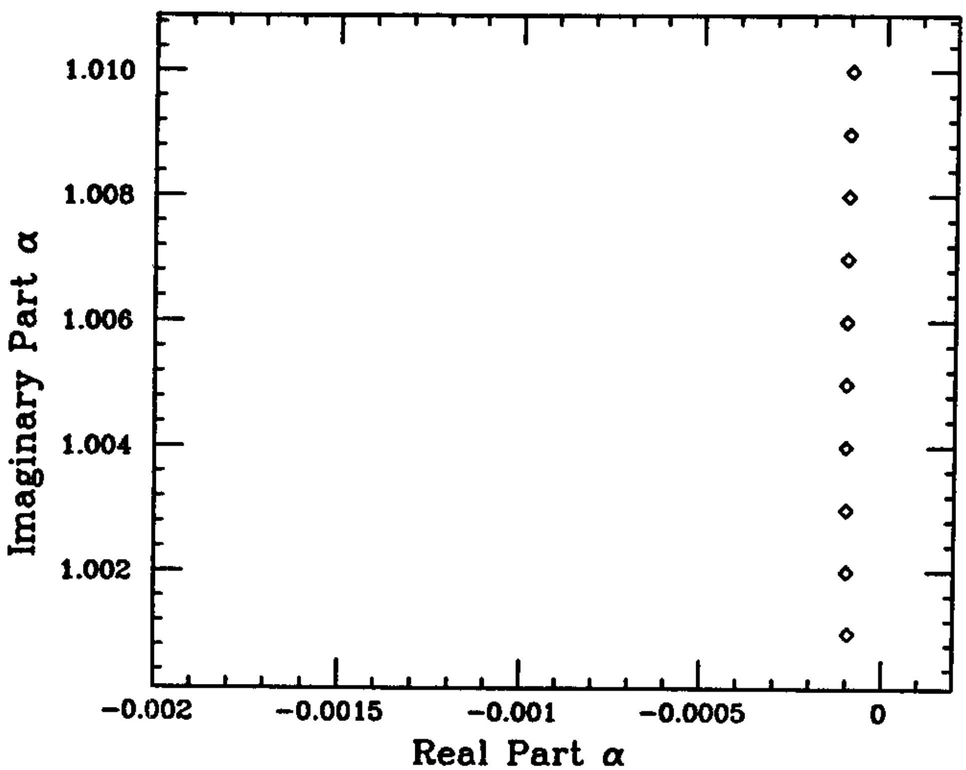

Figure 4-2 Complex frequency spectrum for g=0.0002.

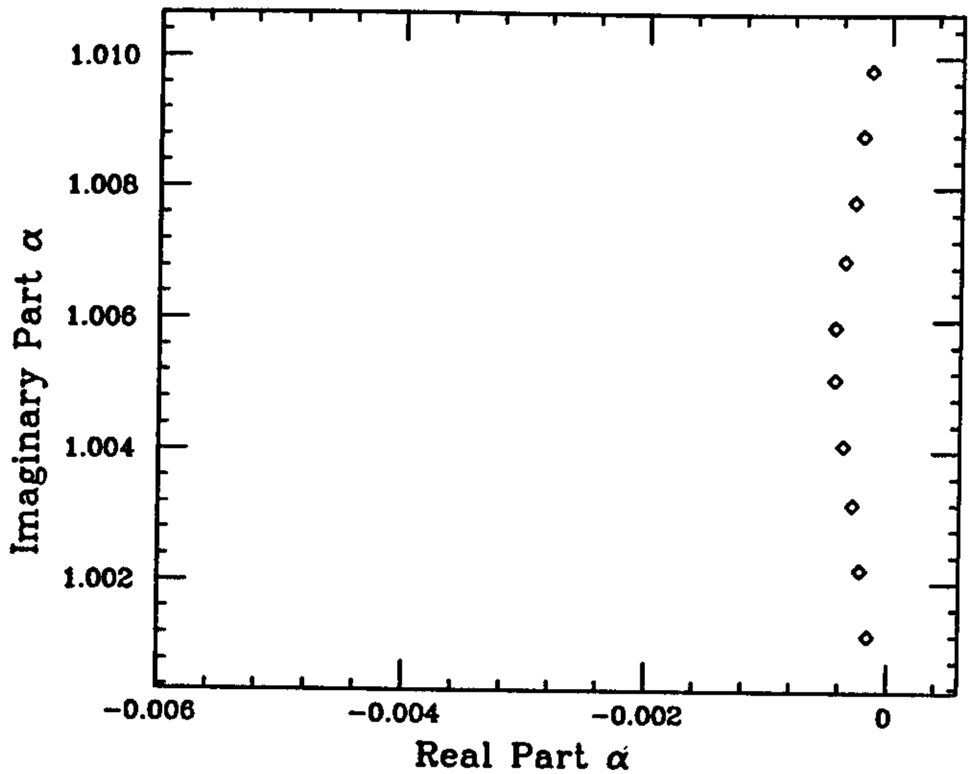

Figure 4-3 Complex frequency spectrum for g=0.0006.

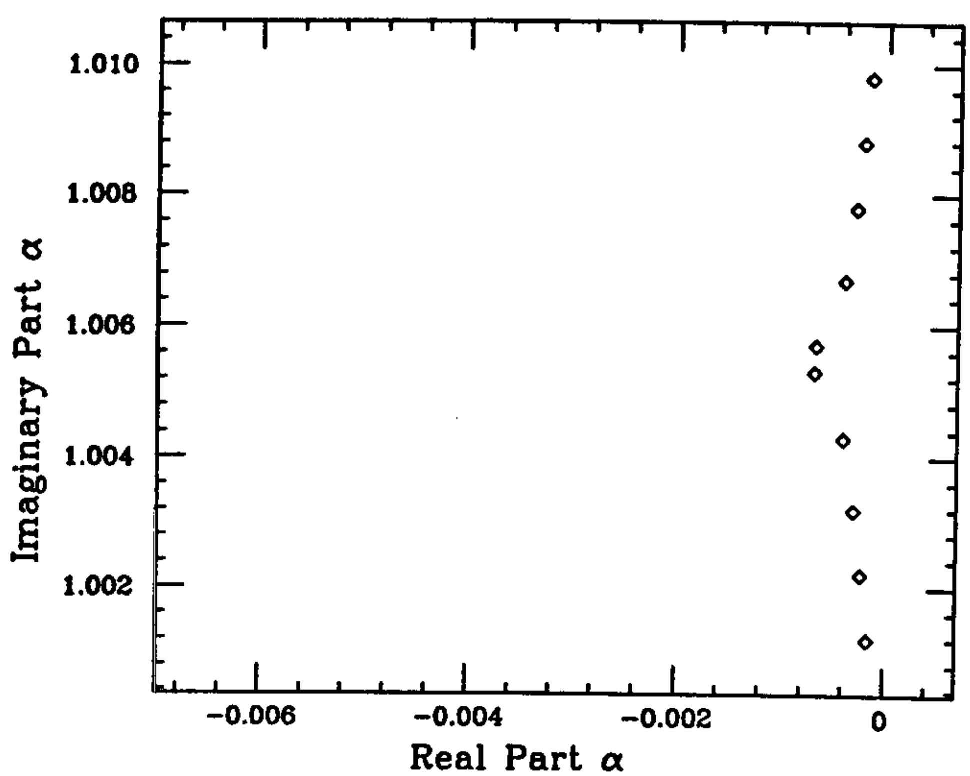

Figure 4-4 Complex frequency spectrum for g=0.0007.

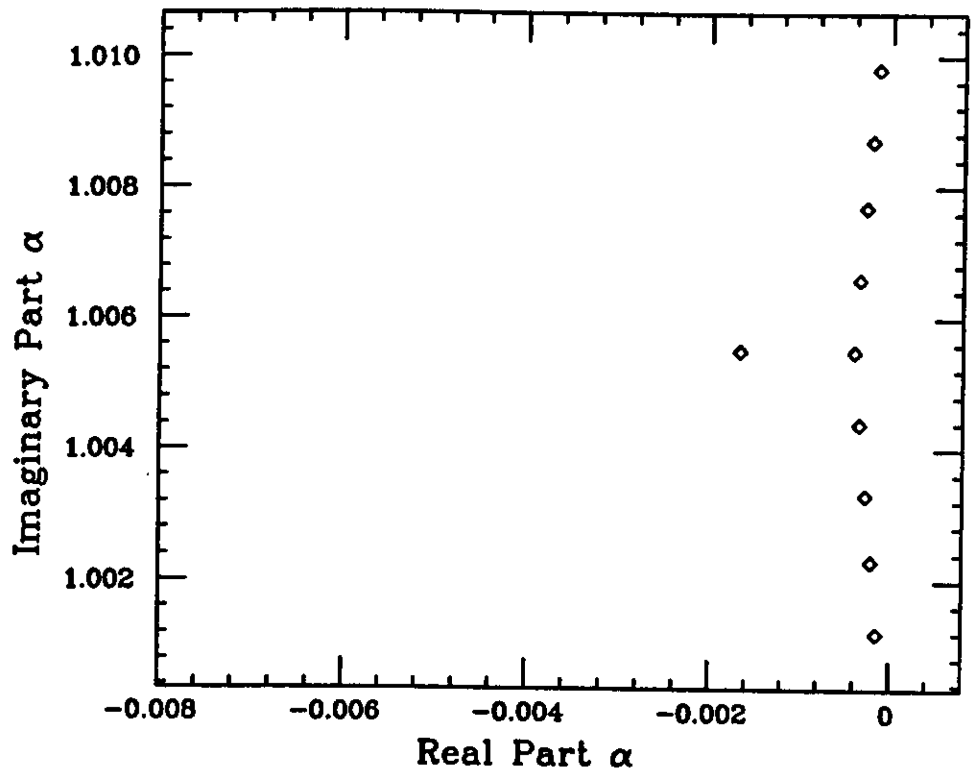

Figure 4-5 Complex frequency spectrum for g=0.0008.

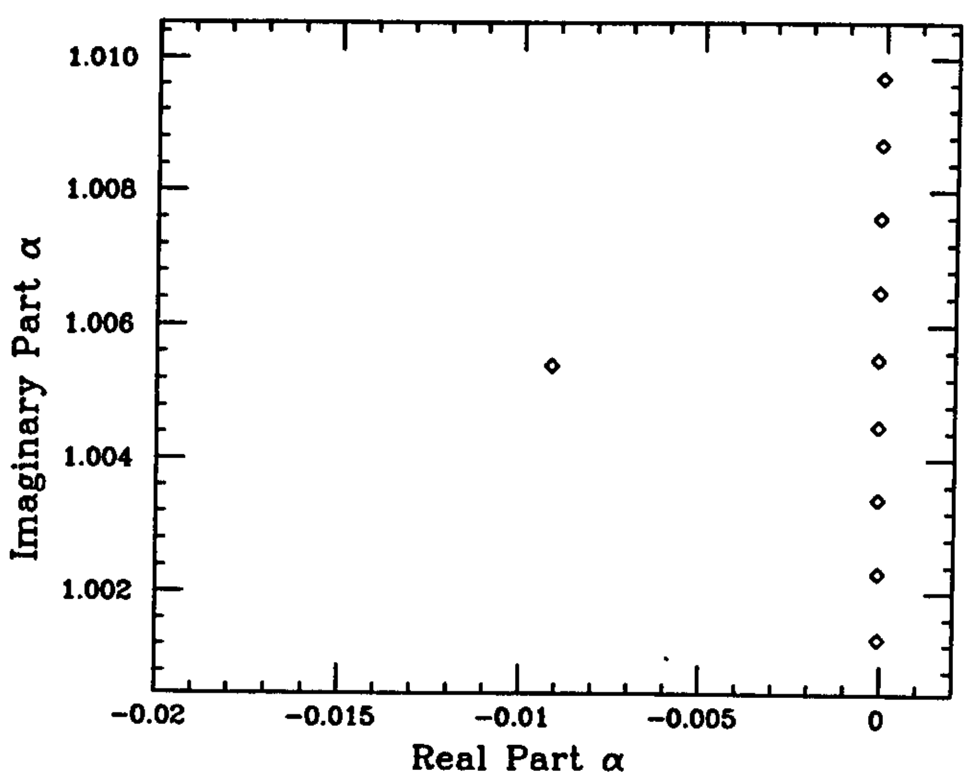

Figure 4-6 Complex frequency spectrum for g=0.002.

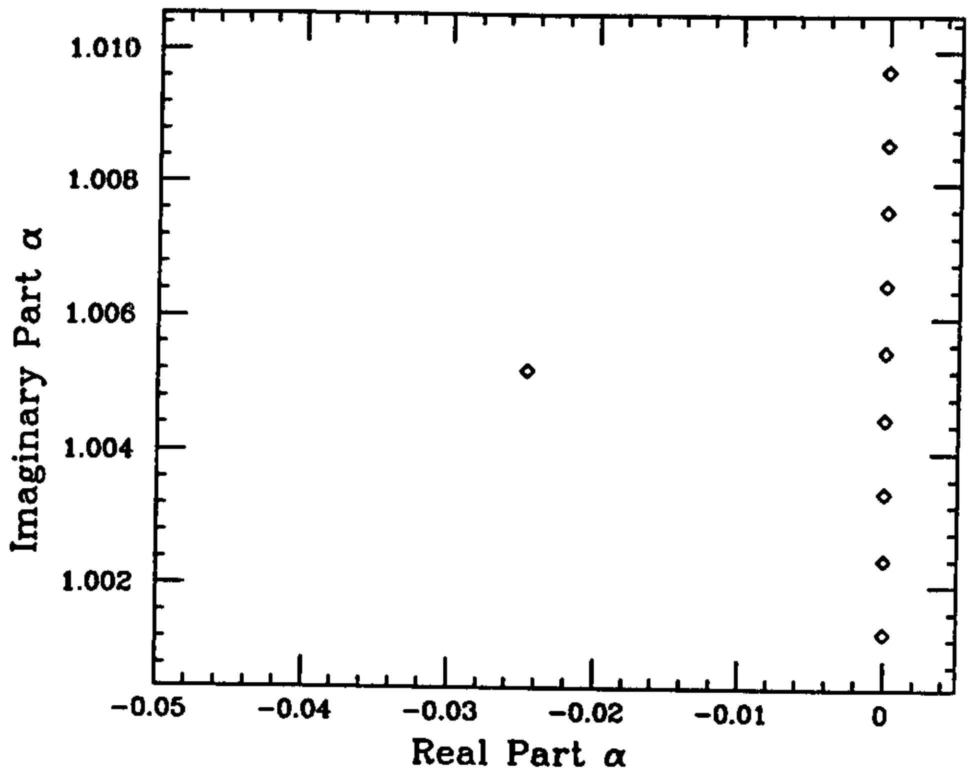

Figure 4-7 Complex frequency spectrum for $g = 0.005$.

that interaction has produced negative correlations among the oscillators' coordinates. In other words, the oscillators have acted in concert to shield the individual fields which produced the damping seen in Fig. 4-2.

Secondly, the correlations became significant only when the damping parameter $g \gtrsim \delta$. If we multiply both sides of this relation by the oscillator number N (in this case $N = 10$), we have a condition on the development of correlations in terms of macroscopic quantities, $N\delta$, the total spread in oscillator frequency, and Ng, the coherent damping rate for the average position of the beam. Thus, correlations induced by a feedback system become significant when

$$\text{coherent damping rate} \gtrsim \text{full frequency spread} \tag{4–5}$$

Finally, if this exericise is repeated with the sign of g changed, essentially the same figures would be found, only with the sign of the real axis changed. Condition (4-5) is maintained with the notion of coherent damping rate replaced by coherent growth rate; that is, there would be a collective oscillation when

$$\text{coherent growth rate} \gtrsim \text{full frequency spread} \tag{4–6}$$

For damping g (e.g., stochastic cooling), condition (4-5) is termed the "onset of bad mixing." The random fields of the particles (the Schottky noise) disappear; there is shielding. This is the direct analogue of Debye screening for the long-range Coulomb force. For unstable g (coherent instabilities), condition (4-6) is described by the notion of Landau damping. "Bad mixing" and Landau damping are, therefore, manifestations of the same basic physical mechanism; coherence can occur only if it proceeds at an underlying rate that is fast compared to the frequency spread.

Between the extremes of $g = 0.0002$ and $g = 0.005$, there is not total damping of the average velocity of the particle. The isolated root does not have a full real part of $Ng/2$ and the nine other roots retain a significant (but diminished) negative real part. If we were to observe the average $\dot{x}$ of the system from turnon, we would first find it at its full random value, which would then damp to a lower value at a rate corresponding to the isolated eigenvalue. From there on the system would "cool" slowly. Exactly the same phenomenon is observed when a stochastic cooling system is turned on. The initial Schottky noise is found to rapidly diminish (Schottky signal suppression), and then the cooling proceeds slowly to diminish the phase space area occupied by the beam. When the suppression is large in magnitude, the rate of suppression is comparable to the coherent damping time of the feedback system.

5. Dielectric Function for a Localized Structure

Now we will proceed with a rather mathematical analysis of the development of correlations for a transverse damping system. The physical model is a treatment of the beam's response to a stimulus as a polarization of a continuous medium. This stimulus may, in fact, be the field of one of the beam particles itself. Thus, there is a mixed description in terms of both the discrete and the continuous. The justification of such a model is found in Bisognano and Leemann [1]. Two important results are obtained. First, the correlational effects are described by a dielectric response function and these correlations exhibit delta-function-like singularity as a function of particle frequencies. Secondly, the scaling of coherent damping and growth to frequency spread as summarized in conditions (4-5) and (4-6) is found to hold in general.

Consider a simple transverse feedback system consisting of a pickup at θ_p and a kicker downstream at θ_k. Let both the pickup and kicker be very short (i.e., approximated by δ-functions). Let $f(a, \phi, \theta, \omega, t)$ be the distribution function for coherent transverse modulations of the beam (a and ϕ are the usual transverse action-angle variables; $x = a \cos\phi, x' = a\nu \sin\phi$) at azimuth θ and revolution frequency ω. The distribution f satisfies the Vlasov equation

$$\frac{\partial f}{\partial t} + \omega\frac{\partial f}{\partial \theta} + \nu\omega\frac{\partial f}{\partial \phi} + F(\theta,t)\frac{\sin\phi}{\nu\omega}\frac{\partial f_0}{\partial a} = 0 \tag{5–1}$$

where F is the transverse kick of the feedback system, and f_0 is the unperturbed distribution, which is assumed to have no ϕ dependence. The kick F is typically of the form

$$F\,(\theta,t) = 2\pi\delta(\theta-\theta_k)\; N\int d\omega' dt'(a'da')d\phi' \;\; G(\omega,t-t')(a'\cos\phi') \;\; f(a',\phi',\theta_p,\omega',t') \tag{5–2}$$

It is assumed that ω is independent of time and the tune ν is independent of ω. Note the δ-function character of the kick and that only values of f at the pickup location enter into the signal. G models the electrical character of the amplifier. After expanding the ϕ dependence of f as

$$f = \sum_{m=-\infty}^{+\infty} f^{(m)}(a,\theta,\omega,t)\, e^{im\phi} \tag{5-3}$$

we have

$$\frac{\partial f^{(m)}}{\partial t} + \omega \frac{\partial f^{(m)}}{\partial \theta} + i\nu\omega m\, f^{(m)} - i\frac{F(\theta,t)}{2\nu\omega}\,(\delta_{m,1} - \delta_{m,-1})\,\frac{\partial f_0}{\partial a} = 0 \tag{5-4}$$

and

$$F(\theta,t) = 2\pi\,\delta(\theta-\theta_k)\,N\int d\omega' dt'(a'da')\,\frac{G(\omega',t-t')}{\nu\omega}\,(a')\,\frac{(f^{(1)}+f^{(-1)})}{2} \tag{5-5}$$

Only the dipole terms $f^{(\pm 1)}$ are excited. If we now Fourier analyze with respect to θ and t, we have

$$\begin{aligned} &i\Omega\, f_\ell^{(\pm 1)} - i\ell\,\omega f_\ell^{(\pm 1)} \pm i\nu\omega\, f_\ell^{(\pm 1)} \mp \frac{i}{4}\frac{\partial f_0}{\partial a}\,e^{i\ell\theta_k}\int d\omega' da' \times \\ &\sum_n e^{-in\theta_p}\left[f_n^{(+1)}(\omega',a) + f_n^{(-1)}(\omega',a')\right]\frac{NG(\omega',\Omega)}{\nu\omega}\,a'^2 + I_\ell^{(\pm 1)}(\Omega,\omega,a) = 0 \end{aligned} \tag{5-6}$$

where $I_\ell^{(\pm 1)}$ is an arbitrary excitation of $f_\ell^{(\pm 1)}$. Let

$$g_\ell^{(\pm 1)} = \int da'\, a'^2\, f_\ell^{(\pm 1)}$$

Eq. (5-6) may then be rewritten

$$\begin{aligned} &i\Omega g_\ell^{(\pm 1)} - i\ell\omega\, g_\ell^{(\pm 1)} \pm i\nu\omega\, g_\ell^{(\pm 1)} \pm \frac{i}{2}\,e^{i\ell\theta_k}\,\bar{h}_0(\omega)\times \\ &\int d\omega' \sum_n e^{-in\theta_p}\left(g_n^{(+1)} + g_n^{(-1)}\right)\frac{NG(\omega',\Omega)}{\nu\omega} + \hat{I}_\ell^{(\pm 1)}(\Omega) = 0 \end{aligned} \tag{5-7}$$

(where $\bar{h}_0 = \int da\, a\, f_0(a,\omega)$) or

$$g_\ell^{(\pm 1)} = \frac{\hat{I}_\ell^{(\pm 1)}}{i(\Omega - \ell\omega \pm \nu\omega)} \pm \frac{i}{2}e^{i\ell\theta_k}\bar{h}_0\int d\omega' \sum_n e^{-in\theta_p}\,\frac{(g_n^{(+1)}+g_n^{(-1)})NG(\omega',\Omega)}{i\Omega - i\ell\omega \pm i\nu\omega}\,\frac{1}{\nu\omega} \tag{5-8}$$

Let $h_\ell = g_\ell^{(+1)} + g_\ell^{(-1)}$. Then we have

$$\begin{aligned} h_\ell = &\sum_\pm \frac{\hat{I}_\ell^{(\pm 1)}}{i\Omega - i\ell\omega \pm i\nu\omega} \\ &+ \frac{i}{2}\,\frac{\bar{h}_0(\omega)e^{i\ell\theta_k}}{\nu\omega}\int d\omega'\,\frac{NG(\omega',\Omega)}{i\Omega - i\ell\omega + i\nu\omega}\sum_n h_n(\omega',\Omega)\,e^{-in\theta_p} \\ &- \frac{i}{2}\,\frac{\bar{h}_0(\omega)e^{i\ell\theta_k}}{\nu\omega}\int d\omega'\,\frac{NG(\omega',\Omega)}{i\Omega - i\ell\omega - i\nu\omega}\sum_n h_n(\omega',\Omega)\,e^{-in\theta_p} \end{aligned} \tag{5-9}$$

Finally, defining $h = \sum_n h_n(\omega', \Omega) e^{-in\theta_p}$, we have the basic relation

$$\int d\omega\; h(\omega,\Omega)\; G(\omega,\Omega) =$$

$$I(\Omega) + \frac{i}{2}\int d\omega\; \frac{NG(\Omega,\omega)}{\nu\omega}\; \bar{h}_0(\omega) \tag{5–10}$$
$$\times \left\{ \sum_\ell \frac{e^{i\ell(\theta_k-\theta_p)}}{i\Omega - i\ell\omega + i\nu\omega} - \sum_\ell \frac{e^{i\ell(\theta_k-\theta_p)}}{i\Omega - i\ell\omega - i\nu\omega} \right\} \int d\omega'\; h(\omega',\Omega)\; G(\omega',\Omega)$$

which yields

$$H(\Omega) = \frac{I(\Omega)}{1 - \frac{i}{2}\int d\omega\, \frac{NG(\omega,\Omega)}{\nu\omega}\, \bar{h}_0(\omega) \sum_\ell \sum_\pm (\pm)\, \frac{e^{i\ell(\theta_k-\theta_p)}}{(i\Omega - i\ell\omega \pm i\nu\omega)}} \tag{5–11}$$

where

$$H(\Omega) = \int d\omega\; h(\omega,\Omega)\; G(\omega,\Omega)$$

and $I(\Omega)$ is an arbitrary excitation at frequency Ω. Schottky signal suppression is described by the denominator of this expression, which we denote by

$$\epsilon(\Omega) = 1 - \frac{i}{2}\int d\omega\; \frac{NG(\omega,\Omega)}{\nu\omega} h_0(\omega) \sum_\ell \sum_\pm (\pm) \frac{e^{i\ell(\theta_k-\theta_p)}}{i(\Omega - \ell\omega \pm \nu\omega)} \tag{5–12}$$

The dielectric response function $\epsilon(\Omega)$ describes the polarization of the beam when there is a localized feedback interaction. Without this particle-particle interaction the response is simply $I(\Omega)$; with the interaction the response is $I(\Omega)/\epsilon(\Omega)$. In other words, the single-particle excitation $I(\Omega)$ is shielded by the other beam particles. Among possible $I(\Omega)$ is the "random" initial excitation of a beam particle; i.e., the unshielded Schottky amplitude. Also note that the $\epsilon(\Omega)$ are expressed in terms of integrals with singular denominators: $i\Omega - i\ell\omega \pm i\nu\omega$. These are interpreted in the sense of an initial value Laplace transform problem. As Ω tends to the real axis there is a delta function contribution, $\Omega = \ell\omega \pm \nu\omega$, and a principle-value integral. The sum over ℓ may be carried out by standard techniques in [4, 5]

$$\sum_\ell \frac{e^{i\ell(\theta_k-\theta_p)}}{i\Omega - i(\ell \pm \nu)\omega} = -\frac{2\pi}{\omega} \frac{\exp i(\theta_k - \theta_p)\frac{(\Omega\mp\nu\omega)}{\omega}}{1 - \exp\frac{2\pi i(\Omega\mp\nu\omega)}{\omega}} \tag{5–13}$$

We note that the sum is discontinuous at $(\theta_k - \theta_p) = 0$, and we have taken $0 < (\theta_k - \theta_p) < 2\pi$ as the natural physical interpretation. For a finite length pickup and kicker, the ℓ sum will be weighted by the Fourier components of the spatial weighting of the kicker and there will be no discontinuity. However, since $\theta_k - \theta_p$ is in practice always greater than the angular extent of the structures, this infinite-sum result should suffice.

The final form of the ϵ-factor becomes

$$\epsilon(\Omega) = 1 - \frac{i\pi}{\omega}\int d\omega\; \frac{NG(\omega,\Omega)}{\nu\omega}\; \bar{h}_0(\omega) \sum_\pm (\pm)\; \frac{\exp i(\theta_k - \theta_p)\frac{(\Omega\pm\nu\omega)}{\omega}}{1 - \exp\frac{2\pi i(\Omega\pm\nu\omega)}{\omega}} \tag{5–14}$$

This ϵ-factor serves two purposes. First, it describes Schottky signal suppression: Schottky power at frequency Ω is modified by a factor $|\epsilon(\Omega)|^{-2}$. Note that the exponential denominator of the integral has poles at $(\Omega \pm \nu\omega) = n\omega$, so that if there is band overlap, there is more than one singularity entering into the integral. Secondly, the condition $\epsilon(\Omega) = 0$ describes the thresholds for coherent motion and instability. As we will see in Section 7, the condition $\epsilon(\Omega) > 1$ for significant polarization (correlations) satisfies condition (4-5), and the $\epsilon(\Omega) = 0$ condition for instability requires that condition (4-6) be satisfied. As we will now see, for sufficiently large gain, there is instability even when the phase of the gain appears appropriate for damping. This effect can be traced to the time delay and δ-function character of the kick, and comes about because of overdamping in a single pass through the feedback system.

Consider a cold beam where $\bar{h}_0(\omega') = \delta(\omega' - \omega)$, $\nu = 0.25$ (plus any integer), $NG = \bar{G}\, e^{i\Omega\tau}$ ($\bar{G}$ is real and τ is the electronic time delay corresponding to $(\theta_k - \theta_p)/\omega$) and there is a one-quarter betatron wavelength between kicker and pickup. Then Eq. (5-14) reduces to

$$\epsilon(\Omega) = 1 + \frac{\pi\bar{G}}{\nu\omega^2}\frac{1}{1 + i\exp\frac{2\pi i\Omega}{\omega}} + \frac{\pi\bar{G}}{\nu\omega^2}\frac{1}{1 - i\exp\frac{2\pi i\Omega}{\omega}} \tag{5–15}$$

or

$$\epsilon(\Omega) = 1 - \frac{2\pi\bar{G}}{\nu\omega^2}\frac{1}{1 + \exp\frac{4\pi i\Omega}{\omega}} \tag{5–16}$$

Setting this equal to zero for coherent modes, we have

$$\exp\frac{4\pi i\Omega}{\omega} = \left(\frac{2\pi\bar{G}}{\nu\omega^2} - 1\right) \tag{5–17}$$

and

$$\frac{4\pi i\Omega}{\omega} = \log\left(\frac{2\pi\bar{G}}{\nu\omega^2} - 1\right) + 2\pi i\, m \tag{5–18}$$

(m any integer) where we have taken the mth branch of the log function. For $1 - \frac{2\pi\bar{G}}{\nu\omega^2} > 0$

$$\Omega = \frac{\omega}{4\pi i}\log\left|1 - \frac{2\pi\bar{G}}{\nu\omega^2}\right| + \frac{\omega}{2}\,(m + \frac{1}{2}) \tag{5–19}$$

For small $\bar{G}$, this yields

$$\Omega = +i\,\frac{\bar{G}}{2\nu\omega} + \frac{\omega}{2}\,(m + \frac{1}{2}) \tag{5–20}$$

The imaginary part of Ω corresponds to damping at $\bar{G}/2\nu\omega$, with a real part at each betatron line. This system provides feedback to damp coherent transverse oscillations. However, when $1 - \frac{2\pi\bar{G}}{\nu\omega^2} < 0$, that is, when there is large damping gain $\bar{G}$

$$\Omega = \frac{\omega}{4\pi i}\log\left|\frac{2\pi\bar{G}}{\nu\omega^2} - 1\right| + \frac{\omega m}{2} \tag{5–21}$$

For $\frac{2\pi\bar{G}}{\nu\omega^2} > 2$ the argument of the log is greater than 1 and we have growth. The frequencies $m\omega/2$ describe neighboring Schottky bands shifting toward each other. This condition

$$\frac{2\pi\bar{G}}{\nu\omega^2} > 2 \tag{5–22}$$

is simply that the coherent damping in one revolution is greater than 2; that is, there is over-damping of coherent motion. With this full-dispersion expression, Landau damping may be included. For an arbitrary frequency distribution a numerical integration of (5-14) will yield precise thresholds of instability. Below the threshold for instability, we may still have $\epsilon(\Omega) < 1$ in the neighborhood of $m\omega/2$. This corresponds to an enhancement of the beam response to external stimulus—for example, amplifier noise. Such a situation is typical of localized feedback systems which produce signal suppression at the centers of Schottky bands and enhancement between Schottky bands.

6. Interparticle Interactions and the Dielectric Response Function

The notion of the dielectric response function is applicable to all sorts of beam interaction. In Section 5, we have derived the dielectric response for a localized, transverse dipole interaction typical of a feedback system. Other interactions of interest include longitudinal feedback systems, transverse and longitudinal coupling impedance (including the space-charge component), and, of course, the full three-dimensional Coulomb interaction of plasma physics. These interactions can have either a local or a distributed character. In each of these cases a dielectric response function can be obtained using techniques similar to that of Section 5. For example, we have transverse the dipole dielectric response (distributed)

$$\epsilon(\ell,\Omega) = 1 - \frac{i}{2}\int d\omega \frac{NG(\omega,\Omega)}{\nu\omega}\bar{h}_0(\omega)\sum_{\pm}(\pm)\frac{e^{i\ell(\theta_k-\theta_p)}}{i(\Omega-\ell\omega\pm\nu\omega)} \tag{6–1}$$

the longitudinal response (localized)

$$\epsilon(\Omega) = 1 + \sum_{\ell}\int dx\, NG(x,\Omega)\,\frac{\partial}{\partial x}\,(\omega f)\,\frac{e^{i\ell(\theta_k-\theta_p)}}{i\Omega - i\ell\omega} \tag{6–2}$$

the longitudinal response (distributed)

$$\epsilon(\ell,\Omega) = 1 + \int dx\, NG(x,\Omega)\,\frac{\partial}{\partial x}\,(\omega f)\,\frac{e^{i\ell(\theta_k-\theta_p)}}{i\Omega - i\ell\omega}$$

and the 3-D Coulomb response (nonrelativistic)

$$\epsilon(\mathbf{k},\Omega) = 1 - \frac{\omega_p^2}{k^2}\int d\mathbf{v}\frac{1}{\mathbf{k}\cdot\mathbf{v}-\Omega-i\eta}\mathbf{k}\frac{\partial f(\mathbf{v})}{\partial\mathbf{v}} \tag{6–3}$$

where ω_p is the plasma frequency: $\omega_p = \sqrt{\frac{4\pi n e^2}{m}}$ for the three-dimensional density n and particle mass m. For the one-dimensional longitudinal systems, $G(x,\Omega)$ describes the rate of change of energy error x per particle at electronic frequency Ω [1].

These dielectric functions describe the deformation of single-particle fields by correlations which develop between beam particles through various interactions. The field at frequency (and spatial harmonic l or wavenumber k for distributed systems) is modified by a factor ϵ^{-1}. The Schottky power density is modified to

$$P(\Omega) \longrightarrow \frac{P(\Omega)}{|\epsilon(\Omega)|^2} \tag{6–4}$$

The associated correlation function can also be expressed in terms of the dielectric response function. Let f_1 and f_2 be the one- and two-particle distribution functions, and define the correlation function g by the decomposition

$$f_2 = f_1 f_1 + g$$

Note that g is a measure of the two-particle correlations that are not due to simple inhomogeneities in the one-particle distribution. For example, particles in a bunched beam have correlated positions even in the limit of no interaction because their azimuth is limited to less than 2π.

For a distributed longitudinal system, the Fourier transforms g_ℓ of the correlation are given by

$$\begin{aligned}\int dx_2\, G(x_2,\ell\omega_2)^*\, g_\ell\,(x_1,x_2) &= G(x_1,\ell\omega_1)^* \frac{f_1(x_1)}{N}\left(\frac{1}{\epsilon(\ell,\ell\omega_1)} - 1\right) \\ &\quad - \frac{1}{|\ell|}\frac{\partial f_1}{\partial x_1}\frac{|G(x_1,\ell\omega_1)|^2}{|\epsilon(\ell,\ell\omega_1)|^2} f_1(x_1)\end{aligned} \tag{6–5}$$

As an example, consider the longitudinal Schottky noise generated by a coasting, uniform beam with a Gaussian energy distribution of width σ_x about some energy E_0. The associated revolution frequency width σ_ω about revolution frequency ω_0, corresponding to E_0, is $\eta\omega_0\sigma_x/\beta^2 E_0$ (where η is the frequency slip factor). At a harmonic n such that $n\sigma_\omega \ll \omega_0$ there will be no significant Schottky band overlap between neighboring harmonics. Without beam self-interaction, the Schottky spectrum P will be given by

$$P(\Omega) = \frac{1}{\sqrt{2\pi}\sigma_\omega}\, e^{-\frac{\left(\frac{\Omega - n\omega_0}{n}\right)^2}{2\sigma_\omega^2}}$$

in the neighborhood of $n\omega_0$ and is illustrated in Fig. 6-1 with the y-scale normalized to unity. A machine impedance Z distributed about a storage ring will produce a dielectric response function of

$$\epsilon(\ell,\Omega) = 1 + \frac{I\beta^2 E_0}{\sqrt{\pi}\eta\omega_0}\,\frac{Z}{\ell}\,\frac{1}{\sigma_x^2}\int dy\,\frac{iye^{-y^2}}{\frac{\Omega E_0\beta^2}{\sqrt{2}\ell\omega_0\sigma_x\eta} - y} \tag{6–6}$$

for beam current I. At frequencies low compared to the cutoff frequency of the beam pipe, the space-charge impedance is given by

$$Z = ikg_0 \tag{6–7}$$

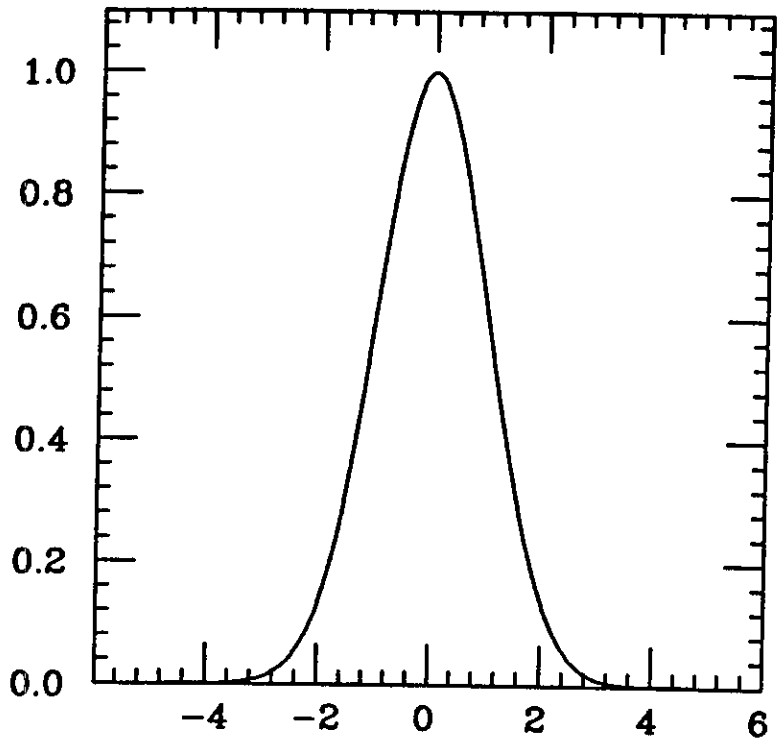

Figure 6-1 Schottky spectrum without space charge.

where g_0 is a constant related to the beam-pipe aspect ratio. Fig. 6-2 illustrates the basic shape of the Schottky band resulting from the dielectric response corresponding to space charge (capacitive impedance) in a storage ring below transition. The effect of an inductive impedance (or space charge above transition) is shown in Fig. 6-3. Fig. 6-4 indicates the Schottky signal deformation for space charge together with wall resistance below transition. Note that there is noise enhancement in Fig. 6-4. This is typical of a system marginally below the threshold for a potential instability. In this case, higher intensity would result in the resistive wall instability. Fig. 6-3 shows that the beam current is nearing the onset of the so-called "negative mass" instability.

7. Harmonic Dependence of Dielectric Response and Screening

The dielectric response function depends strongly on frequency and wavenumber. In turn, the deformation of the signal produced by a perturbation of the beam is a sensitive function of both frequency and wavenumber. As is clear from expressions (5-12) and (6-2), a localized structure mixes together all wavenumbers because of the impulsive character of the interaction. However, note that when there is no Schottky band overlap, a single angular harmonic dominates the expression in the neigborhood of the associated Schottky band. In this situation, the

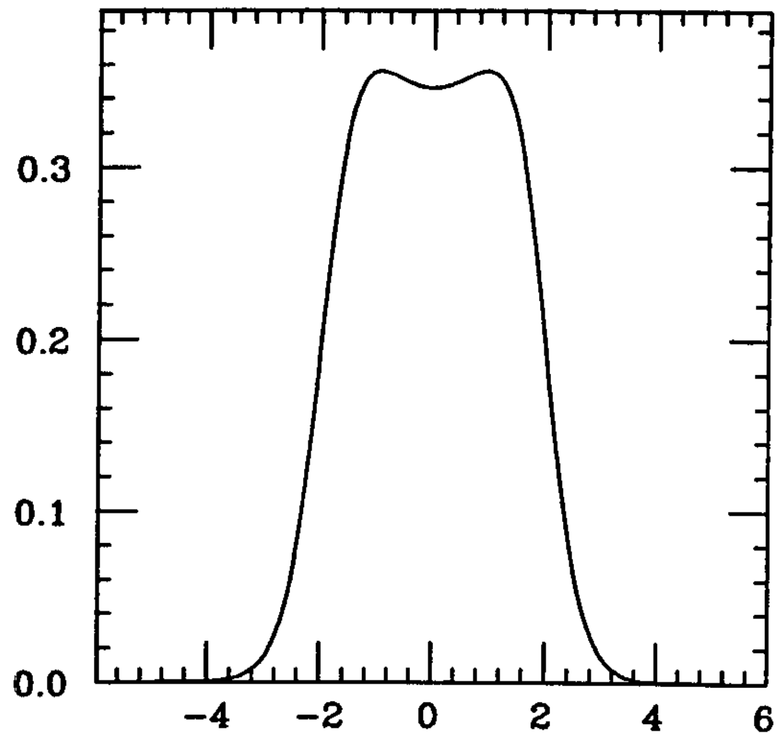

Figure 6-2 Schottky spectrum with space charge and below transition.

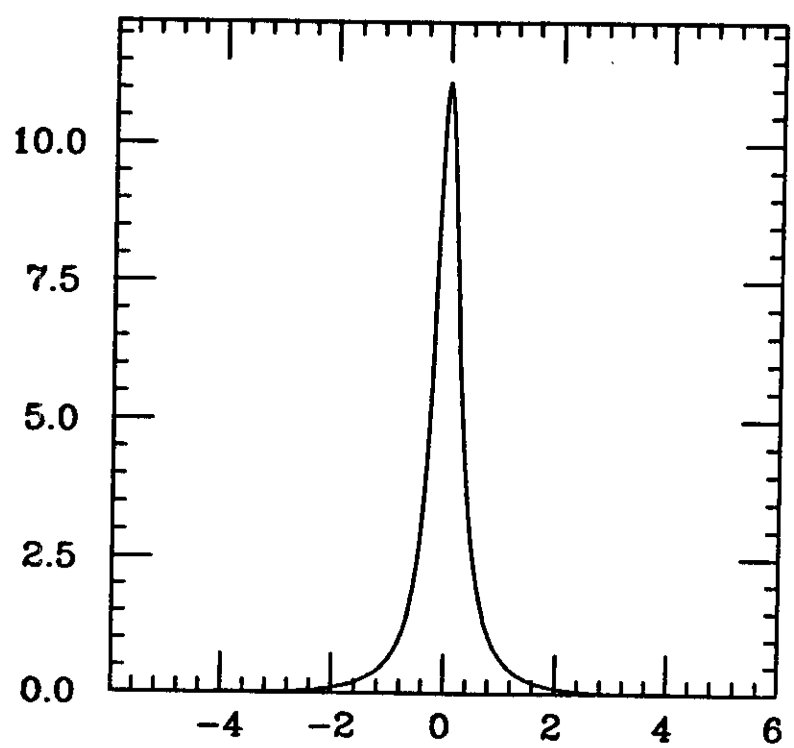

Figure 6-3 Schottky spectrum with space charge and above transition.

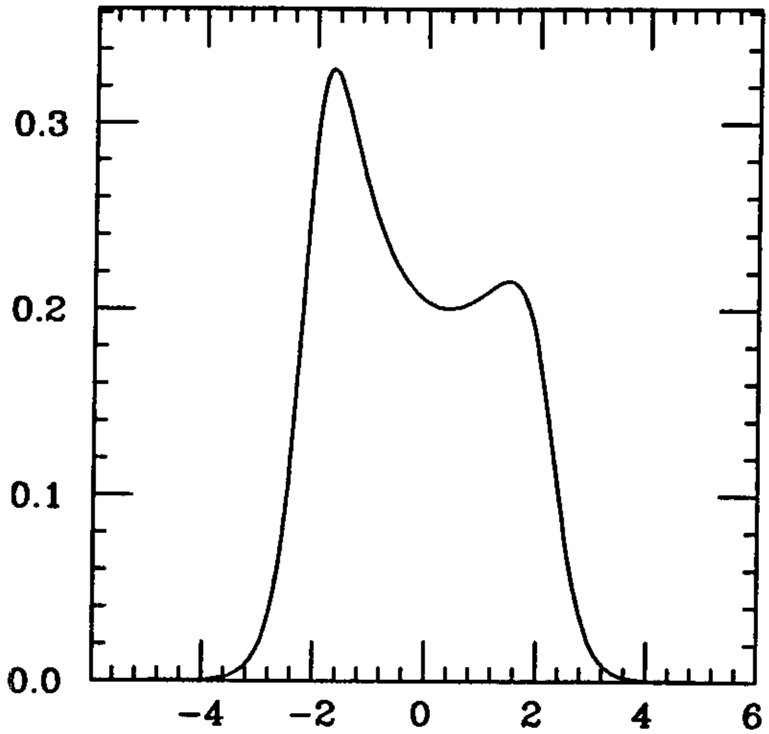

Figure 6-4 Schottky spectrum with space charge and wall resistance, below transition.

expression for a distributed system is a good approximation. For the sake of simplicity (and the immediate analogue to the case of an infinite plasma) we will now focus attention on the ϵ associated with distributed interactions such as wall impedance with a Gaussian energy distribution. At the center of the ℓth Schottky band, the transverse dipole and longitudinal dielectric functions for a storage ring are

$$\epsilon = 1 + \frac{\pi N G(\omega_0, \ell\omega_0)}{2\nu\omega_0 \, |\ell \pm \nu| \, \sigma_\omega} \tag{7-1}$$

and

$$\epsilon = 1 + \frac{N\beta^2 E_0}{|\ell| \sqrt{2\pi}\eta} \, \frac{i}{\sigma_x \omega_0} \left[\frac{G(x_0, \ell\omega_0)}{\sigma_x} \right] \tag{7-2}$$

respectively. For both cases, the magnitude of ϵ is significantly greater than unity when the magnitude of the ratio of coherent damping (or growth) rate to frequency spread is greater than unity. This is identical to that found for the simple 10-oscillator system studied in Section 4. The equivalent expression for the longitudinal dielectric response for an infinite, nonrelativistic static plasma is [6]

$$\epsilon = 1 + \frac{k_D^2}{k^2} \tag{7-3}$$

where

$$k_D = \sqrt{\frac{T}{2ne^2}} \tag{7-4}$$

is the Debye wavenumber for density n and temperature T. Since this plasma is stationary, disturbances with wavenumber k are centered about zero frequency. For a beam moving with velocity v_0 (angular velocity ω_0), the frequency is Doppler shifted up to kv_0 (harmonic $\ell\omega_0$). Thus the beam dielectric functions in Eq. (7-1) and (7-2) are evaluated at $\ell\omega_0$. The Debye wavenumber represents a long wavelength cutoff of fields. The noise produced at wavenumbers below k_D is significantly depressed, and without these fields, the range of the force is effectively reduced. For the particular form of the Coulomb interaction this corresponds to a potential of the form

$$\frac{1}{r}\, e^{-k_D|r|} \tag{7-5}$$

The dielectric response has transformed the k^{-2} Coulomb potential to the $(k^2 + k_D^2)^{-1}$ Yukawa potential.

There is a clear one-dimensional analogue to the classic Debye screening. The one-dimensional Fourier transform of (7-5) is found [7] to be

$$(i/2)\arctan(k/k_D) \tag{7-6}$$

Note that for large k this expression tends to a constant, and for small k it rises linearly from the origin. This behavior is remarkably similar to that shown in Eq. (7-1) and (7-2); that is, for large ℓ the inverse of the response function tends to unity and for small ℓ (assuming G is large enough) the inverse of ϵ is proportional to ℓ. Thus, it appears that these one-dimensional systems characteristic of storage rings also exhibit something like Debye screening.

Because of size limitations, the interaction offered by an rf structure cannot extend to arbitrarily small wavelengths. For nonrelativistic motion, the interaction can extend both behind and in front of the test particle. For relativistic velocities the interaction is only backward, although with time delays, a feedback system can produce both a forward and a backward interaction. As an example, consider a finite, imaginary interaction in wavenumber (the one-dimensional analogue to Coulomb's law) with a flat response up to a rapid short-wavelength rolloff. A typical pulse shape is shown in Fig. 7-1 with the interaction asymmetric in space with respect to the origin. If the low frequencies are cut off with a response function of the form

$$\epsilon(\ell) = 1 + \frac{\ell_D}{|\ell|} \tag{7-7}$$

analogous to (7-1) and (7-2), we find a pulse shape as shown in Fig. 7-2. First note that the peak value of the pulse has been diminished. With finite bandwidth, the dielectric response has cut off a significant portion of the interaction. As is clear from Eq. (7-5), this is not true for truly infinite bandwidth forces where the short-range interaction is not diminished by the shielding. Secondly, note that in Fig. 7-2 the long-range tail of the interaction has nearly vanished. This is the direct analogue of Debye screening. Similar distortions are found for other forms of interaction.

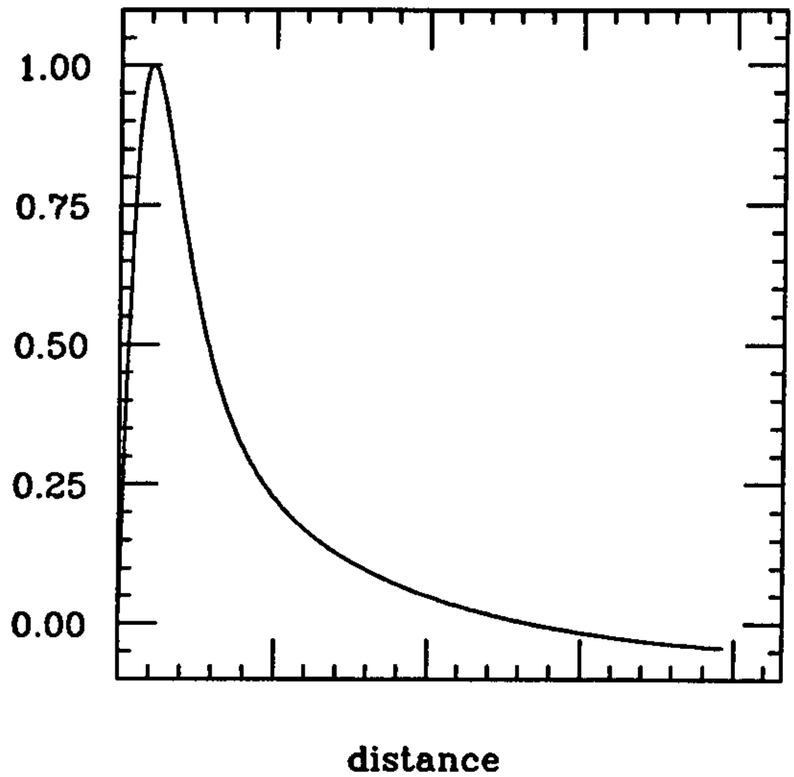

Figure 7-1 Pulse from a finite bandwidth reactive interaction without dielectric shielding.

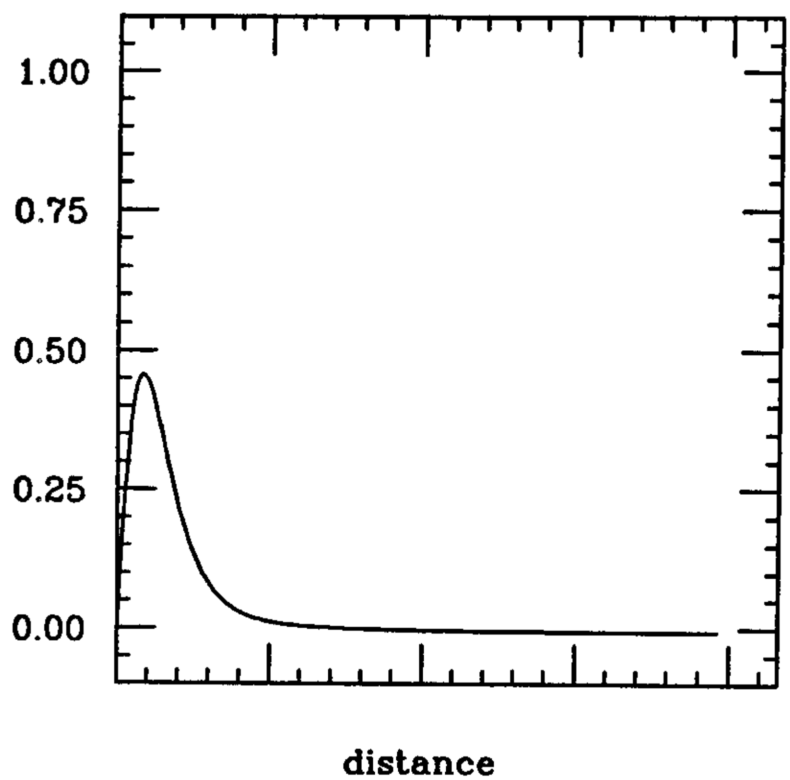

Figure 7-2 Pulse from a finite bandwidth reactive interaction with dielectric shielding.

For this dielectric picture of a continuous medium to make sense, it is necessary that many particles be redistributed with respect to the fields of a test particle. For this to be the case, the Debye screening must not be so severe that only a few particles interact. Thus, for this picture to be physically consistent it is required that for the 3-D Coulomb interaction [6]

$$\frac{n}{k_D^3} \gg 1 \tag{7–8}$$

or

$$\frac{T}{(e^2 k_D)} \gg 1 \tag{7–9}$$

That is, the ratio of the thermal energy to the average interaction strength must be large.

There is a similar small ratio for the one-dimensional storage ring systems which we have focused on here. From Eq. (7-1) we have significant screening (assuming appropriate phases) when

$$\frac{\pi N \left|G(\omega_0, \ell\omega_0)\right|}{2\nu\omega_0 \left|\ell \pm \nu\right| \sigma_\omega} \gtrsim 1 \tag{7–10}$$

that is, when the collective rate is large compared to the frequency width of the Schottky band. For infinite bandwidth, constant G, the condition that the number of particles interacting must be large becomes

$$\frac{N}{R}\left(\frac{R}{\ell}\right) \gg 1 \tag{7–11}$$

or equivalently (for ν negligible)

$$\sigma_\omega \left[\frac{2\nu\omega_0}{\pi \ |G|}\right] \gg 1 \tag{7–12}$$

In other words, if all harmonics $\ell \lesssim N$ are shielded, the interaction is no longer long range with respect to the particle spacing.

It is left to the reader to derive a similar result from Eq. (7-2). Since the frequency distribution width is a measure of the random motion, i.e., temperature, and G is a measure of the particle-to-particle interaction strength, we see that conditions (7-9) and (7-12) are physically analogous. However, there are significant differences. The frequency spread σ_ω is related to the energy spread (and therefore temperature) by the frequency slip factor η ($\sigma_\omega = \eta\omega_0\sigma_x/\beta^2 E_0$). For small η, correlations can become very large through (7-10) even for relatively warm beams. However, as $\eta \to 0$ there is the possibility that (7-12) can be violated. There can be an apparent lowering of the "effective temperature" due to the η factor, which for a storage ring can be considerably less than unity. Finally, for finite bandwidth G, the maximum ℓ with significant gain may satisfy (7-11), and the dielectric picture may have an extended range of validity irrespective of interaction strength.

8. Impact of Gain Shape and Storage Ring Parameters

The Coulomb interaction (nonrelativistic) induces a three-dimensional Fourier field amplitude proportional to

$$\frac{1}{ik} \tag{8–1}$$

Other machine impedance and feedback system gains typically have a faster fall-off. The slow short-wavelength rolloff of Coulomb's law produces the singular $1/r$ behavior on integration over three dimensions. For a feedback system or other resonant interactions with the beam pipe (the wakefield) the interaction remains finite at short distances. With the Coulomb interaction, one can have high enough densities so that the closest particles have interaction energies which exceed the thermal energies; this is the regime of so-called "crystal beams." Because of the repulsive nature of the force, there is a tendency of particles to avoid close coordinates. Typically, machine impedances and feedback systems do not exhibit singular high-frequency (short distance) behavior, nor is the bandwidth sufficient to resolve individual particles. The short-range phenomena are relatively well behaved.

For a storage ring, a small η appears to effectively reduce the temperature of the system. Thus, in the limit of η going to zero it would appear that the screening could become quite pronounced to the point of diminishing Coulomb scattering. However, for suffiently small η, the relaxation from scattering can proceed more rapidly than the development of correlations. Implicit in the analysis presented is that the correlations develop on a time scale that is rapid compared to the changes in the single-particle distribution. Recall the simple oscillator model of Section 4, where the correlations built up N times faster than the single-particle positions damped. This assumption is implicit in the analysis of Section 5, where the single-particle distribution is assumed constant. The same assumption is implicit in all Debye-screening frameworks. For a storage ring the time scale for buildup of correlations scales as $1/\eta$. As η tends to zero, this time scale becomes arbitrarily large. On the other hand, Coulomb scattering occurs on a time scale corresponding to the ratio of mean free path to thermal velocity. This scattering in turn determines the relaxation time of the gross single-particle distribution. For small enough η it is clear that this relaxation will be proceeding faster than correlation will build up, and an analysis in terms of transients is required. Similar remarks are appropriate for transverse motion, where frequency spread due to nonlinearities is usually small compared to the frequency spread induced by longitudinal motion.

Finally, while the longitudinal and transverse coupling G discussed in this paper describe the dominant beam interactions in a storage ring, they represent only the first two terms in an expansion of the forces between any two beam particles. The form of the longitudinal force assumes no transverse variation, and the form of the transverse force applies to a dipole oscillation (a simple transverse displacement) of the beam. There exist higher moment interactions together with corresponding response functions which can be important for high-current beams as found in heavy-ion fusion scenarios. The corresponding collective modes appear as transverse shape and distribution oscillations. This full set of dielectric response functions is the proper analogue of the three-dimensional Coulomb $\epsilon(\mathbf{k}, \Omega)$.

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